Circular Spaces and Privileged Times; What the Game Asteroids Can Teach Us About Presentism

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Abstract

Presentism has long struggled with the results of special relativity. One proposed solution is to stipulate the existence of an ontologically or metaphysically privileged frame which defines the global present for all observers. Recently this proposal has cropped up in literature on spatially closed universes (SCUs) which seem to naturally instantiate such structures. This paper examines the privileged frame proposal through the lens of SCUs, arguing that even in these topologies which seem overwhelmingly friendly to presentism the theory fails. It is then shown how these failures are fundamental to the project, rather than specific to the SCU case.

I'm going to start this paper by asking a rather odd question: What can the game Asteroids, released on Atari over forty years ago, tell us about the nature of time? Surprisingly, the answer turns out to be a lot. In fact, it turns out the game protagonist (a small spaceship), lived in a very strange universe - one where, by simply looking left or right, he would have been able to see not only his past self (including his own birth/construction), but also the founding of his elite asteroid fighting unit, the formation of his solar system, and even the beginning of his universe in the Asteroids big bang. Even more interestingly a number of papers published over the last decade have suggested that he would have been able to locate a ‘privileged’ frame of reference - something philosophers of time have long searched for in our own universe. Indeed, spatially-compact universal topologies (SCUs) turn out to be incredibly attractive for any theory of time that wants to posit an ontologically privileged present precisely because it seems to allow them to escape the problems posed by relativity [Putnam, 1967], and locate an objective privileged ’now’.

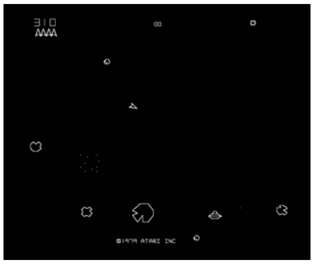

Figure 1: The game of Asteroids, released in 1979 by Atari. It’s universe was spatially compact in all directions - whenever the ship moved off the edge of the screen it would instantly reappear on the opposite side with the same orientation.

This paper aims to critically examine this possibility, working through the mathematics and physics of such universes. Section 1 will lay the groundwork, going over the basics of SCUs and why we might think they generate privileged presents. Section 2 then delves into the problems with the literature. Here I focus on what I take to be the two most damning ones. Firstly, that the results of earlier papers hang on properties of simultaneity in special relativity that aren’t found in general relativity. Secondly, that these privileged frames are non-egalitarian in nature, slaving all other frames to a single dominant foliation. This, in turn, means that the structure has difficulty in grounding the temporal properties of the non-privileged frame. As such, I will argue in Section 3 that not only should the presentist not think that an Asteroids-like universe is friendly to their project, but the problems of the system make approaches like cosmic time, despite their known issues [Ismael, 2016, Callender, 2017], much more attractive in contrast.

For the sake of this paper I will be using, interchangeably, the terms 'presentist', 'A-theorist', and 'tenser' to refer to the theory that 'only present things exist' [Ingram and Tallant, 2018]. By this, I will assume the general stance is that the present is somehow distinguished from the past and future as having some ontological property of 'existing' which defines a universal present. This is in contrast to it's rival theory, the 'B-theory or 'eternalism', which holds that we cannot define an ontologically privileged present and, instead, that terms like 'now' are purely relative in nature, akin to the 'here' for space.

Finally, it is worth noting that while I have jokingly attributed these thought experiments to the fictional world of Asteroids, there are real possibilities that our own universe could have this topology. Spatially-compact topologies have been suggested by string theory [Arfaei and Kamani, 1999], astronomy [Cornish et al., 2004], and dark energy research [Rebouças, 2009], including recent publications from cosmology which suggest that cosmic background radiation supports us living in a spatially closed, rather than open, universe [Di Valentino et al., 2019]. As such, a fair amount might hang on whether such universes are friendly to the presentist project.

1 Closed, Flat Space-Times

Let's start with an overview of the properties of SCUs which make them so interesting. Such universes are spatially flat, finite, and compact, and we're identifying in the same orientation the opposite spatial edges of the universe, meaning it is also non-simply-connected. Time, conversely, is treated as flat and infinite in both directions. We'll use Lorentz coordinates, referring to the universe has having directions $\{t, x_c, y_c, z_c\}$, such that $n_c = [n_0, n_L]$ and $n_0 = n_L$, where $L_n = n_L - n_0$ is the width of the universe in the closed n-th spatial direction.[1] Going forward, for the sake of diagrams we will only depict two of these dimensions, t and x_c. The results that follow will hold in the full four-dimensional regime, however.

As we've already noted, intuitively we can think of this topology as being like that of the game Asteroids, where going far enough in any direction returns you to your original starting point with the same orientation. Note also that our hypothetical universe remains locally euclidean such that locally the standard laws of relativity hold.

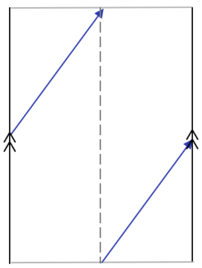

Figure 2: In a closed, flat space-time any observer with motion along the x_c axis (blue line) will eventually return to their original spatial position (dotted line). In this paper identification and direction of identification will be denoted by double arrows. Thus, a rectangle with double arrows on opposite sides, pointed in the same direction is equivalent to a flat cylinder, and a rectangle with double arrows pointed in opposite directions denotes a mobius strip.

Consider a pair of inertial observers within this system (we will take the traditional Alice and Bob but in sticking with the theme feel free to imagine them as living in small triangular spaceships). We now stipulate that Alice is stationary with respect to the x_c direction, such that her worldline progresses in a straight line up our diagram, while Bob is moving along the x_c dimension such that his worldline is slanted and he will eventually 'loop' around the known universe returning to his original spatial position (Figure 2).

The first interesting property of this Asteroids-universe is that we are able to easily distinguish between Alice and Bob with a simple test: Imagine Alice sending out a pair of light-signals in opposite directions along the x_c axis. These signals travel around the universe, move across the identified edges, and return to Alice simultaneously. Bob can perform

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[1] It should be noted here that the sudden discontinuity in x-coordinates is a remnant of the metric we are using, and is no more significant than the fact that people standing on the line of zero longitude on our own planet seem to be able to step from 0° to 360°.

the same test. For him, however, when he releases the pair of signals the system is no longer symmetric - he is moving away from one ray of light and towards another. As such, one ray will return sooner than the other (Figure 3). Thus, we are able to distinguish between frames which are stationary with respect to the background space-time and frames which are moving. Moreover, we are able to determine with what speed our non-stationary frames are moving along x_c, and in what direction.

Peters (1983) further notes that non-comoving observers in this universe will measure its width (L_x) to have different values. Consider two identified points on the opposite 'edges' of our universe for Alice, $A_1(t_A, x_A) = (0, 0)$ and $A_2(t_A, x_A) = (0, L_x)$, where ($A_1 = A_2$). Consider, now, Bob's frame. How wide, we can ask, is the universe from his perspective? As our system is locally minkowskian, we can use Lorentz boosts to figure out the answer by converting coordinates A_1 and A_2 into Bob's frame.

In Bob's frame we can contrive it such that $B_1 = A_1 = (0, 0)$. It turns out that B_2, however, comes out to be $(-v\gamma L_x, \gamma L_x) \neq A_2$. That is, where Alice sees the universe as L_x wide, Bob sees the universe as γL_x wide, where γ is the standard Lorentz factor and v is his speed in the x_c direction. As γ is always greater than or equal to one, this means that Alice, as the 'stationary' observer, will always calculate the smallest universal width when compared to her counterparts moving with non-zero velocity in the x_c direction.

As an immediate implication of this, while Alice identifies $(0, 0)$ with $(0, L_x)$, Bob, conversely, will see a universe that identifies $(0, 0)$ with $(-v\gamma L_x, \gamma L_x)$. This means that not only is the universe narrower, from Bob's perspective, but he identifies the two sides at an angle (Figure 4b)

It is here we start to see why the presentist might think that the Asteroids-universe is friendly to their project - the angled identification has important implications for the way Alice and Bob carve the universe into past, present, and future using simultaneity relations. For now we will follow the Weeks (2001) and Peters (1983) in using radar synchrony to construct these lines of simultaneity, but we will return to this assumption in Section 2.1 as part of our discussion of the central problems with this type of presentist project. For the moment, however, we will set these worries aside and progress apace with the rest of the literature on the subject.

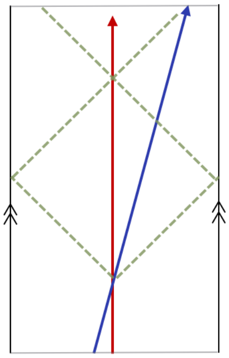


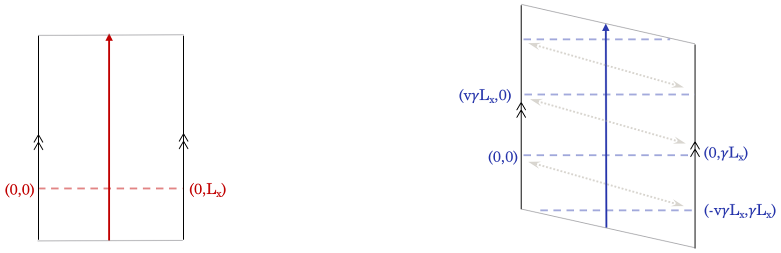
Figure 3: Alice (red) and Bob (blue) will meet light signals (green) at different points depending on how fast they're moving relative to the x_c direction.

Radar synchrony (also known as Einstein Synchronisation) involves calculating what events are simultaneous to an observer at time τ_1 by imagining the observer emitting a light-signal aimed at a given event. This signal 'bounces off of' the event, and is returned to the observer, who receives it at time τ_2. By assuming the speed of light is constant the observer can calculate at what time between τ_1 and τ_2 the 'bounce' happened in their own frame (call this τ_3) using equation 1. Thus, the points τ_3 and the reception/emission point for the event are radar simultaneous, where,

$$\tau_3 = \tau_1 + (\frac{1}{2}\tau_2 - \tau_1) = \frac{1}{2}(\tau_1 + \tau_2) \qquad (1)$$

In a compact universe, this is obviously streamlined by the fact we no longer need a 'bounce', instead observers can simply send out a single signal and receive it themselves at a later point once it has 'looped' around the universe.

Using radar synchrony for distances less than L_x for Bob and Alice will give us the standard results we see in flat, non-periodic topologies. When we expand to considering simultaneity for points with distances greater than L_x, however,



(a) Alice's worldline (solid red) with Alice's calculated plane of simultaneity at time t=0 (dashed)

(b) Bob's worldline (solid blue) with Bob's calculated plane of simultaneity at time t=0 (dashed). Note that from Bob's perspective the universe is both wider, and identified at a diagonal.

Figure 4

Alice and Bob get startlingly different results. For Alice, her identification of $(0,0)$ with $(0,L_x)$ means that her calculated plane of simultaneity will wrap around the universe, meeting up and overlapping with itself. This structure will cut her universe in two - everything in her forward lightcone is in the future and backwards lightcone is in the past (Figure 4a). The diagonal identification we calculated above for Bob, however, gives him a strange result. Since Bob identifies $(0,0)$ with $(-v\gamma L_x, \gamma L_x)$, as his line of simultaneity crosses the identification point it ‘jumps’[2] up the temporal dimension (Figure 4b). Geometrically, mapping both of these to a single diagram, we can read this as Bob's plane of simultaneity angling such that it wraps around the universe infinitely in both directions (Figure 5).

This line of simultaneity for Bob also intersects his own past and future lightcones an infinite number of times. Recall the definition of a Cauchy surface as one such that every inextensible, non-spacelike causal curve intersects it exactly once. In this sense, Bob's ‘present’, unlike Alice's, is not a Cauchy surface as it

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[2] I am hesitant to use this language as it implies there is something special about the points of identification within the universe. Recall, however, that this is a remnant of the metric being used, and the location of the discontinuity is a matter of convenience and choice rather than a real break in the world.

might have been in a non-compact system (Figure 6). Moreover, it fails to divide the universe in half - there is no decisive distinction between the past, present, and future for Bob-like observers according to radar synchrony. For Bob, then, he views the present as a strange beast - he is simultaneous with both himself 'now', and his older self, and his older older self, etc etc. as well as his younger self, younger younger self, and so forth.[3]

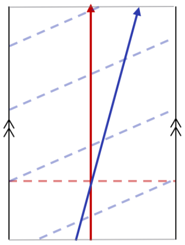

Figure 5: Alice's calculated simultaneity plane at time t (red) and Bob's calculated simultaneity plane at time t (blue) graphed together in Alice's frame of reference.

We might wonder exactly how this works. After all, relativity tells us that if Bob looks like he's moving along x_c relative to Alice, then from Bob's perspective Alice is the one moving, and he remains still. This is true in open universes where the systems are symmetric. In a closed universe, however, as Weeks (2001) notes, the symmetry is broken by the connected nature of space-time. Put simply, the fact that observers can tell whether they're stationary with respect to x_c or not alters the dynamics of the system.

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[3] It is worth noting that even though we have a system with significant restrictions on the cauchy surfaces available, there are still multiple viable cauchy surfaces for Alice's frame. slice up the universe such that each world-line only cuts every surface once. 'Wavy' surfaces, for example, are still valid solutions provided they never have an angle greater than 45° from the x_c direction (Figure 7).

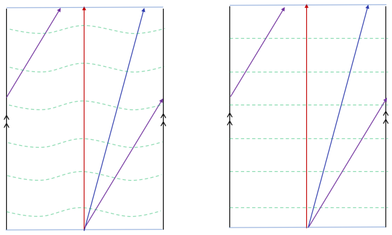

Figure 7: Both figures depict viable cauchy slicings (green dashed lines) of the universe for a topologically compact space-time. World-lines (Red, Blue, Purple) will only ever cut each surface once.

1.1 A Privileged Present?

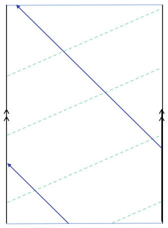

Figure 6: This figure can be read either as visualising a single cauchy surface which wraps around the universe, or as multiple parallel cauchy surfaces at an angle. In either case, the world-line depicted cuts the surface multiple times.

Let us now turn to why we might think that these types of global structures support the existence of a privileged present. The focus in the literature is on Alice's frame for obvious reasons - every other frame shares the same strange properties as Bob, giving us spiralling planes of simultaneity, widely variable universe widths, and an inability to cleanly divide past from future via simultaneity with the present. Alice, conversely, has a uniquely well-behaved present to which we can ascribe the following properties,

1. Identifiable - Alice's frame, and thus radar synchronus plane of simultaneity, is able to be objectively located by all observers as being stationary relative to the space-time topology.

2. Splits the universe into two regions – Alice's plan of simultaneity, unlike non-Alice frames, cleanly separates the global past from the global future

3. Unique - if time t_1 is the present, then it is not the case that time t_2 is also the present, where $t_1 \neq t_2$.
4. Universal - all events will agree on what other events are or are not in Alice's defined present.
5. Derivable from/compatible with relativity - We derive the unique properties of Alice's frame from our most fundamental theory of space-time, rather than independently of, or in spite of them.
6. Productive/is a cauchy surface - Alice's plane of simultaneity slices the entire space and is able to generate both past and future states.
7. Couched in fundamental physics vocabulary - The definition of Alice's present doesn't rely on averages or concepts that are otherwise statistically defined.

Bob's calculated planes of simultaneity, conversely, fail on points 2 and 6, as will all other frames that aren't comoving with Alice's one.

This list is a highly attractive set of properties if we're looking for an ontologically privileged present. Alice's calculated plane of simultaneity seems to be able to do the work needed of a 'present' in ways that no other frame in her system can. Importantly, it escapes one of the major pitfalls of modern presentism - it is not only compatible with relativity, but the uniqueness appears to actually fall directly out of the theory. The incompatibility of presentism and relativity is notorious, and despite many attempts to reconcile the two it remains one of the dominant problems for the field. It is astounding, then, that something that looks like a solution comes out of something as simple as identifying two sides of a finite universe.

2 The Problems with Presentism

2.1 Defining Simultaneity

All these properties are why many papers in the literature talk about Alice's frame as 'preferred' in some way. Here, for example is Peters (1983),

> "Periodic boundary conditions applied to the space-time of special relativity lead to the existence of a preferred time." (p.6)

and Luminet (2009),

> "As we explain below, all the inertial frames are not equivalent, and the topology introduces a preferred class of inertial frames." (p.18)

Certainly it is undeniable that she is distinguished from all other non-comoving observers in the universe by the symmetry breaking that occurs in this topology. As we've already hinted at, however, there is a fundamental problem with many of the conclusions above - namely those that result from the use of radar synchrony to define planes of simultaneity for Alice and Bob. This reliance on radar synchrony is based on a common assumption that there exists a coherent definition of simultaneity for relativity, something that is categorically false. In reality radar synchrony is, at best, a useful heuristic for a limited set of cases with very specific properties including those covered by special relativity - itself a limiting case of general relativity. Any attempt to read from these results to broader conclusions about the fundamental structure of space-time is doomed to failure, however, by the non-general nature of the results.

Jammer (2006) has a nice derivation of this result, which we will draw on here. Without going too deep, we know that in general relativity when we try to apply radar synchrony we get that two clocks C_a at location A and C_b at location B are 'simultaneous' when

$$C_A = x^0 + \frac{1}{2}(dx^0_{BA} + dx^0_{AB}) \tag{2}$$

where dx^0_{AB} and dx^0_{BA} are the coordinate times taken for a light signal to travel from point A to B and from B to A respectively.

With some more trivial mathematics, we can calculate the difference in readings between clocks C_a and C_b to be,

$$\Delta x^0 = -\frac{g_{0\alpha}}{g_{00}} dx^\alpha \tag{3}$$

where $\alpha, \beta = (0, 1, 2, 3)$, and $g_{\alpha\beta}$ is the standard metric tensor defined across a Lorentzian manifold M.

This equation allows us to synchronise between to points that are infinitesimally close to one another in a way that is consistent with general relativity, essentially calculating simultaneity along paths. Note, however, that only when $\Delta x^0 = 0$ does radar synchrony work, otherwise there is a difference of Δx^0 between two events that are supposedly 'simultaneous'. That is, the assumption that we can use equation 2 to determine what counts as simultaneous with a given point in a given frame relies on the zero value of $g_{0\alpha}$. Importantly then, equation 3 only disappears when we're considering open curves, and thus static metrics.

Special relativity is, of course, build around a static metric. We normally express it using standard minkowskian structures, where $ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2 - c^2(dx^0)^2$ and $g_{00} = -1, g_{11} = g_{22} = g_{33} = 1$ and all other $g_{0\alpha} = 0$. Hence, $-\frac{g_{0\alpha}}{g_{00}} dx^\alpha = 0$ and radar synchrony holds.

Outside of specially selected cases, however, the metric we consider in relativity is almost never static. Indeed, only in very rare limiting cases will radar synchrony give us anything approximating a useful simultaneity relation. In all other contexts general relativity not only denies that this method works, but is such that no two points are consistently simultaneous *from within the same frame of reference* - the calculation of simultaneity about a closed curve will leave us with clashing results between our initial measurement and our final if $g_{0\alpha} \neq 0$. This is especially true once we allow for the existence of objects like gravitational waves which deform the metric itself over time.

Synchronisation in general relativity has long been known to have these types of troubles. Reichenbach (1951), and Landau and Lifshitz (1951), noted the problem with $g_{0\alpha}$ in 1951, as do Grünbaum (1969), and Rizzi et al (2004) amongst others, pointing out that the reason for these conflicting results is that radar synchronisation is simply a widely adopted convention - not something fundamental to the theory itself. Indeed, simultaneity relations can never be derived from general relativity as the theory is only able to define ordering relations between causally connected events. That is, events in the forward lightcone of an event happen after it, events in the backwards lightcone happen before it. As the lightcone is restricted to containing time-like separated events, and simultaneity relations are between space-like separated events, we are unable to treat simultaneity using the structures of general relativity. This is the cause of the discrepancy between Bob's strange simultaneity relations, and yet perfectly standard causal relations - the latter is fundamental to the space-time structure, the former is not.

This means that when physicists and philosophers like Weeks and Luminet use Bob's strange measurements of the present to try and justify Alice as a naturally privileged frame, they are relying on a system which is, at best, a handy tool with no physical meaning, and at worst a system which fails to hold in almost all real-world cases.

The fact that radar synchrony is simply an arbitrary convention should come as no surprise. Consider, for example, what would happen once we allowed our SCUs to contain matter, and thus space-time curvature. The path of light is warped by the presence of such phenomena, and as such even outside of the convention arguments simply throwing a blackhole anywhere near the Alice and Bob systems would wildly alter Alice's calculated planes of simultaneity and thus her apparently nice results.

The use of radar synchrony to try and define an objective, ontologically privileged temporal property of the universe is akin to attempting to derive fundamental results about time from Newtonian mechanics. One cannot simply assume that a result that derives from a limit of a theory holds more broadly.

Given this there is little reason to think that there is anything significant about the strange simultaneity results that seem to tell between Alice and Bob's frames for the presentist. Instead, the apparently strange results of Bob are just that - odd aberrations that hinge on an arbitrary choice of simultaneity convention rather than a significant result with genuine meaning for our understanding of time.

2.2 Equal Access Time

The problem of simultaneity is particular to the types of derivations being done for SCU universes. It raises the spectre of larger problems for the project, however.

Consider the fact that one of the reasons Alice seems to have such nice properties for presentist accounts of time is the fact that we are picking a single privileged foliation based on one dominant frame of reference - Alice's. In contrast, we are effectively slaving all other frames to the privileged one - Bob gets no say in what counts as the present, nor can he locate the present within his own frame without first boosting his observations to Alice's frame of reference.

This brings with it serious concerns. Firstly, now that we have shown that one of the properties that was most heavily leant on - Alice's unique slicings of the universe - isn't fundamental to the system, it's unclear how exactly Bob's present is related to Alice's one. What does it mean to say there is an objective present for Bob when we are unable to say at at what time Bob is simultaneous with Alice and thus in the objective present?

Secondly, and perhaps more damningly to the presentist project as a whole, we run up against a common problem for any attempt at defining a universal present by picking a single preferred time, namely how on earth this frame imposes it's properties on the rest of the universe's observers. Bob, as we've already discussed, has a very different view of the present to Alice. What the SCU-presentist proposes, however, is that Bob's experience of time (and in particular his 'now') is entirely dependent on properties that are only found in Alice's frame of reference.

Perhaps even more difficultly, the presentist seems left needing to explain how the global structure of space-time impacts Alice and Bob's experiences of the world. How can topological features that may be entirely inaccessible to both Alice and Bob in their lifetimes, provided the L_x is large enough, reach in and affect their local presents?

In this way one of the properties of SCU-time that initially made it so attractive - the existence of a single privileged frame - has suddenly become one of its biggest weaknesses. Because Bob cannot access the ontologically privileged present himself, and Alice can no longer define it for him, we are left with a system that seems to abandon Bob and anyone else who isn't co-moving with Alice's frame.

Another way of putting this is to realise that in the spatially compact case it seems to be necessary that Alice's frame have some special connection to the phenomenal experience of temporal passage - something Bob must, conversely, lack, or which he only has as a derivative of Alice's frame. How the universe recognises this frame is left unanswered, though, nor is it clear how Alice's frame can impact Bob's lived experience. Other less problematic attempts to cache out the 'flow' of time generally rely on local mechanisms, including statistical mechanics and temporal asymmetry, which are at least causally immediate to the observers in question, even if, again, it remains unclear how the entire universe manages to do so in sync. Either way, the presentist seems left needing to explain how the global structure of space-time impacts Alice and Bob's experiences of the world.

3 Cosmic vs. SCU time

These problems become even more stark when we compare SCU times with another approach to melding relativity and presentism: Cosmic time. Swinburne (2007), in particular, has a nice example of that will work well as a contrast class. On his approach an observer attaches to each galaxy a fundamental frame, centred on the galaxy's average motion of matter. These frames, in our own universe at least, appear to be accelerating ones - curving outwards from a point in the early universe when these galaxies were relatively close to one another, and (at least in broad strokes) within the same inertial frame. Under the assumption that the universe is isotropic, and that the laws of physics are universal and thus that clocks work the same way in every fundamental frame, we can trace every accelerating frame back to this point in the early universe. At this moment all fundamental clocks can be synchronised, either by slow-clock transport when the galaxies were close together, or by recognition of the shared inertial frame. Let this moment when all fundamental clocks could be synchronised be a globally agreed upon 'first tick' in time. Any observer, in any galaxy, can make measurements from within her own fundamental frame and trace back how long it's been from their perspective since this original tick. This, then, is the universal clock, where global time is defined locally by the time since the 'first tick', and planes of simultaneity are defined across the fundamental frames through agreements in time since said tick.

The nice thing about these types of cosmic time approaches is that they are able to be located from within every frame: Both Bob and Alice can make a measurement, from within their own universe's fundamental frame, and come up with a universal 'time'. Cosmic time is an equal opportunity clock – neither Bob nor Alice are privileged over the other, but both have equal claim to finding the privileged present. This, in turn, allows Swinburne's Cosmic Time to avoid all of the problems proposed above. Because Alice, Bob, and all other observers are able to locate the privileged present from within their own frame we no longer need to worry about how Alice can dictate the present in the absence of a simultaneity relation. Indeed, we no longer need a simultaneity relation at all to have an ontologically privileged present - Bob and Alice are not simultaneous because of a defined relation between their frames, but because of a property inherent to the frames themselves. In this, approaches like Swinburne seem far more attractive - they constitute a more universal world-time, one that doesn't simply pick out a single observer as special while ignoring all other observers.

Cosmic Time does, obviously, have its well-known problems [Callender, 2017, Ismael, 2016]. Certainly it is not, itself, a clear solution to the presentist project. In contrast to SCU time, however, it is clear that approaches like Swinburne's do have their advantages though, advantages that help highlight why the A-theorist ought to be cautious about taking SCUs to be the silver-bullet they initially seemed to be.

Conclusion

We started this paper with the rather ridiculous question of what we could learn about time from the game Asteroids. What we see, having worked in some small way through the dynamics of such universes, is that the experience of any pilot in the game would actually be a very strange one, full of wrapped light-cones and varying universe widths. It is undeniable that this makes compact space-times interesting and worth further investigation. Importantly, however, and contrary to what other authors have suggested, it is not the case that such universes are obvious candidates for having a privileged frame of reference of the kind desired by presentists.

Recall our earlier list of desirable properties for the 'now',

1. Identifiable
2. Splits the universe in two
3. Unique
4. Universal
5. Derivable from/compatible with relativity
6. Productive/is a Cauchy surface
7. Couched in fundamental physics vocabulary

Of this list, the kinds of presents supposedly found in compact space-times tick all of these boxes, while cosmic time, notoriously, fails spectacularly at number 7 and number 3 given the arbitrary choices that seem to go into defining the 'global' frame. Let us revisit this list with a more critical eye, though. In the literature philosophers and physicists relied heavily on requirement 3 to justify thinking of Alice as the privileged observer. She seemed to generate the only physically reasonable simultaneity slicing for the universe, and thus was the best (and only) choice for defining a global now.

As we've just seen, however, these calculations relied on the faulty assumption that simultaneity results from special relativity hold more broadly in general relativity. Once we look deeper, however, it becomes clear that these don't translate into the broader theory. Indeed, general relativity, in contrast to special relativity, doesn't allow us to generate objective simultaneity relations between two events. Instead, it can only distinguish the ordering of two events that are time-like separated. Given this, the results from Weeks, Luminet and others may hold in highly restricted cases, but seem unable to tell us anything foundational about time in general.

Moreover, this result also throws a spanner into the works for number 5 as Alice's privileged present suddenly seems to rely heavily on special, rather than general, relativity, and thus cannot be said to be genuinely consistent with relativity as a whole, any more than we can take results from Newtonian physics to hold more broadly at a relativistic scale.

Secondly, we might argue that, given the apparent benefits of cosmic time over spatially-compact time, we should add an eighth criteria,

8. Plays the time role for all relevant observers.

which spatially-compact time fails to meet.

The Asteroids Topology provides us with a nice playground within which to show the broader problems of presentist approaches that attempt to slave our concept of the present to a single frame of reference and the failures we see here are ones that can be generalised to any such attempt, regardless of topology. As we are reminded by our toy model, general relativity is fundamentally unfriendly to the concept of objective simultaneity. This is the spectre haunting any attempt to raise a particular frame to king, and slave all others to it, and one that even the Asteroids universe, which seemingly gives the presentist so much of what they wish for, cannot escape.

Bob has legitimate reasons to be sceptical about the claim that Alice is ontologically privileged. Certainly it is the case that Alice has unique properties, and it is undeniable that compact space-times are interesting and strange. If nothing else the fact that we can objectively measure the motion of any observer relative to the background space-time is an intriguing result that may have deep implications for the way we think about the reality of space-time itself. In general, however, it does not seem that this result translates into a privileged present in a way that should convince either us or the Bob's of the world that we should think of Alice as a natural choice for an ontologically privileged present.

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